HYDRODYNAMIC CHARACTERISTICS OF FLOW IN SYSTEMS WITH TURBINE IMPELLERS*

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The study deals with a phenomenological description of a flow leaving the blades of different types of turbine impellers, which are always placed in a cylindrical vessel with radial baffles under a turbulent regime of liquid flow. The flow description is determined by three basic characteristics of the proposed model: *i.e.* the diameter of a cylindrical tangential jet a, the width parameter of a cylindrical tangential jet σ and the parameter of the intensity of momentum flux in the flow leaving the turbine A. All these parameters were calculated from the measured profiles of mean velocity for different types of turbines (standard, closed and with profiled blades) in systems of various dimensions. From the results follows that for geometrically similar agitated systems the model parameters have the same values; they vary with the type of turbine used and are independent on the relative size and position of the impeller in the system.

The description of flow in the stirred system primarily depends on the description of the flow which leaves the blades of the rotating impeller and is the source of energy and motion of the whole system. Therefore, this part of the system has been given attention in many studies. Several mathematical models were also published, aiming at a description of flow in this region[1-3]. These models contain parameters which require an experimental determination. Fořt and coworkers[4] found the values of these parameters by measurement in a cylindrical system with a standard turbine impeller. It may be assumed that the determined values are applicable to systems with similar geometry.

The studies quoted consider a stirred system with a turbine impeller placed in its axis (Fig. 1). At the vessel wall there are four radial baffles reaching from the bottom to the liquid surface. The flow leaving the blades of the impeller has a conical shape and is determined by the coordinates r, z, α_0 (Fig. 2). The origin of the coordinate system is in the plane of impeller separating disc. The axis z coincides with the axis of symmetry of the vessel. The angle α_0 is formed by the vertical plane of measurement and by the plane of the two opposite baffles. The direction of mean flow is

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determined by the angles α and β. The angle α is formed by the mean velocity vector $\bar{w}$ and by its projection in the axial-tangential plane. β is the angle between the mean velocity vector and its projection in the radial-tangential plane. The mean velocity components are then $\bar{w}_r$, $\bar{w}_{tg}$, and $\bar{w}_{ax}$. The description of the velocity field in the jet streaking from the region of the rotating turbine impeller is based on the continuity equation and Reynolds equation[2,3] for the mean flow under the turbulent regime of stirred liquid. The assumption involved is that the turbine impeller as a source of motion may be replaced by an axisymmetrical cylindrical slot – tangential cylindrical jet (Fig. 2). The liquid streaking from such jet has its non-zero mean velocity component only in tangential direction and flows into the liquid of an infinite volume which is at rest.

THEORETICAL

The analytical solution[2,3] of the continuity equation and Reynolds equation for the mean flow under turbulent flow regime of stirred liquid leads, after introducing

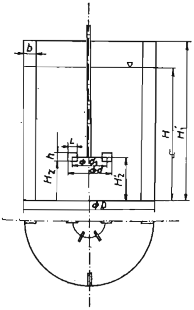

FIG. 1
Stirred system with turbine impeller

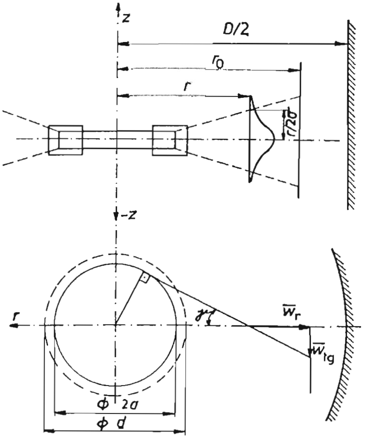

FIG. 2
Cylindrical axisymmetrical tangential jet

eight simplifying assumptions and two suitable boundary conditions, to the three--parameter equation

$$\overline{W}_r = \frac{A}{2\pi d^2 h}\left(\frac{\sigma}{r^3}\right)^{1/2} (r^2 - a^2)^{1/4}\left[1 - \mathrm{tgh}^2\left(\frac{h\sigma}{4r}\right) Z\right] \tag{1}$$

expressing the dependence of the radial mean velocity component $\overline{w}_r$ on the axial coordinate Z (Fig. 2), and to the relations for the axial ($\overline{w}_{ax}$) and tangential ($\overline{w}_{tg}$) mean velocity components

$$W_{tg} = W_r \, \mathrm{tg}\, \alpha \,, \tag{2a}$$

$$W_{ax} = 0 \,, \tag{2b}$$

where

$$\alpha = \arcsin\,(a/r)\,, \tag{3a}$$

$$\beta = 0\,. \tag{3b}$$

The dependent and independent variables are expressed in the dimensionless form

$$W_j = \overline{w}_j/(\pi\, dn)\,; \quad [j = ax, r, \mathrm{tg}]\,; \tag{4a}$$

$$Z = 2z/h \tag{4b}$$

suitable for modelling purposes, *i.e.* the mean velocity components are related to the peripheral velocity of impeller blade tips πdn and the axial coordinate is related to the half-height of blade $h/2$. The parameter a characterizes the so-called diameter of a cylindrical tangential jet (Fig. 2), which may be determined from the angle of mean velocity direction in the investigated liquid jet[3] (see Eq. *3a*). The parameters σ (parameter of width of the investigated jet) and A (parameter of the intensity of momentum flux in the jet) can be determined from the measured velocity profiles $W_r = W_r(Z)$. The considered phenomenological model allows also the determination of eddy viscosity ε, based on the knowledge of the three above parameters in the jet streaking from the rotor region of the turbine impeller

$$\varepsilon = \frac{A}{2}\frac{1}{(\sigma^3 r)^{1/2}}\frac{2r^2 - a^2}{(r^2 - a^2)^{3/4}}\,. \tag{5}$$

All the mentioned dimensional and dimensionless characteristics are to be investigated in their dependence on the system size, the turbine type, the relative impeller-to-vessel diameter ratio d/D and the relative position of the impeller in the system given by its height above the vessel bottom H_2/D, or H'_2/D. We may assume that the influence of the simplex d/D on the parameter values will not be apparent, unless the flow from the rotating impeller is, immediately after its origination, influenced by the vessel wall or baffles, *i.e.* if the impeller rotates in an "unlimited" space. The influence of H_2/D, or H'_2/D will occur, if the impeller is at an appreciable distance from the geometrical centre of the vessel, *i.e.* if $H'_2/D \ll H/2D$. In the case of geometrically similar systems different in size we may infer that the dimensionless characteristics of the investigated flow will also be independent of the value of the characteristic dimension of the system, but they will markedly change with respect to the used impeller type.

EXPERIMENTAL AND CALCULATIONS

The measurement was carried out[5] in a cylindrical flat-bottomed vessel of diameter $D = 400$ mm (Fig. 1), provided with four radial baffles of width $b = D/10$ at the wall. The used liquid was water filled up to the height $H = D$. Four types of turbine impellers in five geometrical systems were employed (Table I).

The mean velocity in the flow leaving the blades of the turbine impeller was measured by the DISA hot-film anemometer. Two ways of measurement were used: the measurement with a rotating probe, which does not register the distorsion caused by the rotation of impeller blades, and the

TABLE I

Investigated types of systems with turbine impeller

System	Type of turbine impeller	Range of $Re_M \cdot 10^4$	d/D	H'_2/D	Note
S_1	standard (Rushton)	1·48–5·6	$\frac{1}{3}$	$\frac{1}{2}$	$d : h : L : d_1 =$ $= 20 : 4 : 5 : 15$
S_2	standard (Rushton)	1·48–4·7	$\frac{1}{3}$	$\frac{1}{3}$	$d : h : L : d_1 =$ $= 20 : 4 : 5 : 15$
S_3	closed (see below)	1·48–6·5	$\frac{1}{3}$	$\frac{1}{2}$	see Fig. 3
S_4	closed (see below)	1·48–6·5	$\frac{1}{3}$	$\frac{1}{2}$	see Fig. 3
S_5	with profiled blades	2·2–6·5	$\frac{1}{4}$	$\frac{1}{2}$	see Fig. 4

System	d mm	d'_1 mm	d_2 mm	h_1 mm	h' mm	h'_1 mm	g mm
S_3	133	98	80	22	13	3	1·5
S_4	133	98	80	10	7	2	1·5

measurement with a stationary probe. The radial coordinate of the measuring point r was always at the distance of 7 mm from the impeller blade tip, whereas the z-coordinate was varied in the range $z \in \langle -13\cdot3; +13\cdot3 \text{ mm}\rangle$, from which always 10 values were chosen. The range of impeller rotational frequency of revolutions $n \in \langle 50; 220 \text{ min}^{-1}\rangle$.

The experiments results are represented in the form of axial profiles of the quantities α and $\bar{w}$ (or $\bar{W}_r$) related to a radial distance by 7 mm longer than the impeller diameter $d/2$ at various geometrical and physical conditions of the system (Table I). From the profiles $\alpha = \alpha(z)$, $[r = \text{const.}]$ the values of the quantity a were calculated by means of the relation

$$a = r \sin \alpha_{av} , \qquad (6)$$

where the value α_{av} is the arithmetical mean (average value) of the measured values of the angle α in the given profile. The measured values did not almost differ for the given radial coordinate. The quantity $\bar{w}_r$ was calculated for the values of $\bar{w}$ and α by means of the relation

$$\bar{w}_r = \bar{w} \cos \alpha \qquad (7)$$

and normalized by the value of the peripheral velocity of impeller blades. The shape of the axial profile of the dimensionless radical velocity component was then considered in the form

$$W_r(z) = A_1\{1 - \text{tgh}^2 [A_2(Z - A_3)]\} , \quad [R = \text{konst.}] , \qquad (8)$$

the dimensionless axial and radial coordinates in this relation being defined either by (*4b*) or by

$$R = (r - a)/r , \qquad (9)$$

TABLE II

Parameters of velocity field in flow leaving the region of rotating turbine impeller

System	α_{av}	a mm	$2a/d$	σ	$s(\sigma)_{rel}$	$A/(2\pi d^2 n)$	$s(A/(2\pi d^2 n))_{rel}$
S_1	45	52·1	0·78	17·9	9·4%	0·122	8·6%
S_2	42	49·3	0·74	15·3	4·3%	0·120	2·8%
S_3	26·8	32·3	0·50	12·5	3·7%	0 035	14·6%
S_4	41	48·2	0·72	11·4	9·0%	0·103	9·1%
S_5	44	51·0	0·93	18·8	5·1%	0·070	2·4%

in which the already calculated value of the parameter a for the given flow pattern had been substituted. The parameters A_1, A_2 and A_3 in Eq. (4) were determined from the experimental data by means of non-linear regression analysis, the closeness of correspondence between the courses of the experimentally found and calculated profiles $\overline{W}_r = W_r(Z)$ having been evaluated statistically.

The parameter A_1 in Eq. (8) represents maximum value of the radial component of liquid velocity in the given profile normalized by the peripheral velocity of impeller blades. It may be expressed as [see Eq. (1)]

$$A_1 = (A/2\pi\, d^2 n)\,(\sigma/r^3)^{1/2}\,(r^2 - a^2)^{1/4}\,. \qquad (10)$$

The parameter A_2 equals to the reciprocal width of the jet of liquid flow in the given point $\sigma/(2r)$, normalized by the half-height of impeller blade $h/2$, because

$$A_2 = \sigma h/4r\,. \qquad (11)$$

The parameter A_3 in Eq. (8) expresses the dimensionless axial coordinate of the velocity profile maximum, *i.e.* its displacement with respect to the horizontal plane z (or Z) = 0. The value of this parameter was found to be statistically insignificant (the above described processing of experimental data had proved that in all the experiments the A_3 values did not exceed the order of hundredths). The axial profiles

TABLE III

Parameters of velocity field in flow leaving the region of standard (Rushton) turbine impeller

Author	H_2'/D	d/D	σ	$A/(2\pi d^2 n)$	D m
Drbohlav[7]	$\frac{1}{3}$	$\frac{1}{4}$	13·16	0·092	1·0
Drbohlav[7]	$\frac{1}{3}$	$\frac{1}{3}$	15·35	0·140	1·0
Bertrand[5]	$\frac{1}{3}$	$\frac{1}{3}$	15·27	0·120	0·4
Möckel[8]	$\frac{1}{2}$	$\frac{1}{4}$	13·77	0·115	0·5
Möckel[8]	$\frac{1}{2}$	$\frac{1}{3}$	17·15	0·093	0·5
Bertrand[5]	$\frac{1}{2}$	$\frac{1}{3}$	17·87	0·122	0·4
Möckel[8]	$\frac{1}{2}$	$\frac{2}{5}$	14·85	0·112	0·5
Möckel[8]	$\frac{1}{2}$	$\frac{1}{2}$	14·60	0·106	0·5
Average values			*15·25*	*0·113*	

of all mean and fluctuating characteristics of flow leaving the impeller region are then symmetrical along the horizontal plane which includes the impeller separating disc.

The quantities σ [see Eq. (*11*)] and A, or – in dimensionless form – $A/(2\pi d^2 n)$ [see Eq. (*10*)] were calculated from the non-linear regressions of A_1 and A_2 for the known values of the parameter a and the radial coordinate r (or R). In this way the characteristic parameters of the considered flow were acquired and given further treatment: From the quantities σ and $A/(2\pi d^2 n)$ the mean values and their relative standard deviations for the given geometrical conditions were calculated. The values of these quantities are listed in Table II. This Table also contains the calculated values of a in a dimensionless form, *i.e.* $2a/d$, expressing the relative dimensions of the cylindrical tangential jet with respect to the impeller diameter.

DISCUSSION

The processed velocity measurement results prove that the proposed model is adequate for a description of the velocity field in the flow from the turbine impeller blades. The comparison of the results by different authors (Table III) treated in the same way shows that the dimensionless parameters σ and $A/(2\pi d^2 n)$ may be con-

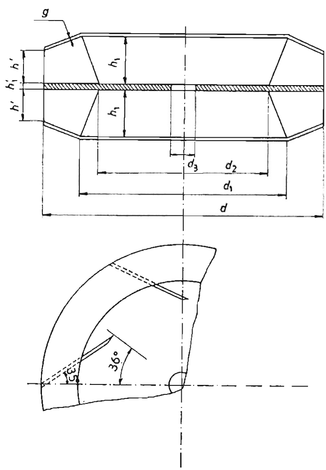

FIG. 3
Closed turbine impeller

sidered independent of the size of the investigated system, as well as – in the measured range of the geometry simplexes d/D and H_2'/D (or H_2/D) – even on these parameters. A considerable fluctuation of the values of σ and $A/(2\pi d^2 n)$ around their means may be attributed to a certain degree of their dependence on the value of the radial coordinate R [see Eq. (9)], although theoretically they should not have been subject to any such change. The found values of the dimensionless parameter of the radius of the cylindrical tangential jet $2a/d$ (Table II) correspond to the published equation[4]

$$2a/d = 2{\cdot}087\ \mathrm{Re_M}^{-0.106}\,, \tag{12}$$

where the mixing Reynolds number is defined as follows

$$Re_M = nd^2/\nu\,. \tag{13}$$

The values of all the three parameters of the investigated flow for the employed nonstandard types of turbine impellers (Table II and Figs 3 and 4) differ from the values measured with a standard[9] (Rushton type) rotor (Table III). Especially the types S_3 and S_5 (relatively higher closed turbine and turbine with profiled blades) represent different sources of streaming (tangential slots) with the radius a. Also the shape of flow jet from these sources differs from the standard configuration: the jet

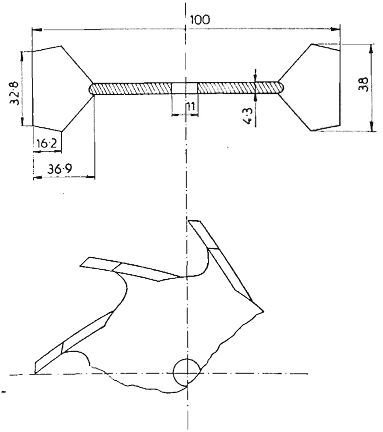

FIG. 4
Turbine impeller with profiled blades (dimensions in mm)

from the nonstandard turbines is narrower. Yet even in these cases the proposed phenomenological description of the flow in the jet streaking from the turbine impeller may be considered adequate with respect to the accuracy of determination of the discussed parameters.

LIST OF SYMBOLS

A	parameter of momentum flux intensity in flow leaving turbine impeller, $m^2 s^{-1}$
a	diameter of cylindrical tangential jet, m
b	baffle width, m
D	vessel diameter, m
d	impeller diameter, m
d_1	diameter of impeller separation disc, m
d'_1	diameter of stator, m
d_2	diameter of inner circle circumscribed by impeller blades, m
d_3	diameter of impeller hub, m
g	thickness of impeller blade, m
H	still height of liquid level in vessel, m
H_2	height of lower edge of impeller blade above vessel bottom, m
H'_2	height of impeller disc above vessel bottom, m
h	impeller blade width, m
h'	height of lower (upper) part of impeller blade, m
h'_1	height of impeller disc, m
L	impeller blade length, m
n	impeller rotational frequency, s^{-1}
R	dimensionless radial coordinate
r	radial coordinate, m
Re_M	mixing Reynolds number
$s_{(t)rel}$	relative standard deviation of quantity t
W	dimensionless velocity
w	velocity, $m s^{-1}$
$\overline{w}$	mean velocity, $m s^{-1}$
Z	dimensionless axial coordinate
z	axial coordinate, m
α	angle formed by mean velocity vector $\overline{w}$ and by its projection in axial-tangential plane, deg
β	angle formed by mean velocity vector $\overline{w}$ and by its projection in radial-tangential plane, deg
ε	eddy viscosity, $m^2 s^{-1}$
σ	parameter of width of cylindrical tangential jet
ν	kinematic viscosity, $m^2 s^{-1}$

Subscripts

ax	axial
r	radial
tg	tangential

REFERENCES

1. Cutter L. A.: AIChE J. *12*, 35 (1966).
2. De Souza A., Pike R.: Can. J. Chem. Eng. *46*, 64 (1968).
3. Drbohlav J., Fořt I., Krátký J.: This Journal *43*, 696 (1978).
4. Fořt I., Möckel H. O., Drbohlav J., Hrach M.: This Journal *44*, 700 (1979).
5. Bertrand J.: *Thesis*. Institut du Génie Chimique, Toulouse 1980.
6. Bertrand J., Couderc J. P., Angelino H.: Chem. Eng. Sci. *35*, 2157 (1980).
7. Drbohlav J.: *Thesis*. Prague Institute of Chemical Technology, Prague 1976.
8. Möckel H. O.: *Thesis*. Ingenieur Hochschule Köthen, Köthen 1978.
9. Czechoslovak Standard. Research Institute of Chemical Engineering – CHEPOS, Brno 1969.

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